

Computer Science 360
Final Examination
(Open Text Books and Notes)

Time: 3 hours
Marks (Total 100)

December 11, 2002

- 8 1. Given that the language $L = \{a^n b^n | n \geq 0\}$ is not regular
- Give an example regular language L' such that $L \cup L'$ is regular.
 - Give an example regular language L'' such that $L \cup L''$ is not regular.
- 12 2. For each of the following statements indicate whether it is true or false. Give a reason for each decision.
- Some problems in NP have lower bounds of $\Omega(2^n)$.
 - The halting problem is in NP .
 - The problem of determining whether or not an undirected graph is biconnected is in P .
 - Satisfiability is in NP .
 - The branch-and-bound technique cannot be used on NP -complete problems.
 - Every problem in NP is NP -complete.
- 12 3. Construct a pushdown automaton for the language
- $$L = \{a^n b^m b^p c^p b^{n-m} | n \geq 0, m < n, p > 2\}$$
- 12 4. The 3SUM problem is to determine whether or not there are three elements of a set S of n integers that sum to zero. Give an $O(n^2 \log^2 n)$ time divide-and-conquer algorithm for the 3SUM problem.
- 12 5. Given a directed graph $G = (N, A)$ and a set of nodes $Q \subset N$, give a very efficient algorithm to determine whether or not each node in $N \setminus Q$ can reach a node in Q via a directed path.
- 14 6. Define the 360-encoding, $E(w)$, of a string $w \in \{a, b\}^*$ as follows: $E(e) = e$, where e is the empty string, $E(a) = 360$, $E(b) = 063$, and if $w = cu$ where $c \in \{a, b\}$ and $u \in \{a, b\}^*$ then $E(w) = E(c)E(u)$. For example $E(aba) = 360063360$. Define the 360-encoding of a language $L \subseteq \{a, b\}^*$, $E(L)$, to be the set of 360-encodings of the strings in L .
- Prove that if $L \in \{a, b\}^*$ is accepted by a finite automaton, then $E(L)$ is regular.

- 12** 7. Let A be an array of n distinct integers. An inversion in A is a pair of integers x and y such that x appears before y in A but $x > y$. Describe an $O(n \log n)$ time divide-and-conquer algorithm for determining the number of inversions in A .
- 18** 8. In an undirected graph $G = (V, E)$, a set $S \subseteq V$ is an independent set iff no two vertices in S are adjacent in G . An interval graph $H = (V, E)$ is the intersection graph of a set I of intervals of the real line. That is $V = I$ and two vertices are adjacent iff their corresponding intervals intersect. Give an efficient greedy algorithm for finding the maximum cardinality independent set of vertices in an interval graph. Assume you are given the set I of intervals. How much time does your algorithm require? Prove that your algorithm is correct.